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Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

HOLE-HOLE INTERACTIONS AND THE PROPERTIES OF NUCLEAR MATTER

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Recently a number of authors[1,2,3] have suggested modifications of the Brueckner theory of nuclear matter[4] so as to include hole-hole interactions, as well as particle-particle interactions. Iwamoto[2] has demonstrated that in a perturbation theory calculation the inclusion of hole-hole interaction makes no change in the ground-state energy through second order. The singular two-body potential between nucleons makes it difficult, however, to conclude anything about the contribution of these terms in nuclear matter. The formal similarity between the equation of Iwamoto and the equation for the energy gap in nuclear matter[5], coupled with the fact that the energy gap is very small at normal density[6], indicates that the effect of hole-hole interactions is probably only a

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† Supported in part by the Office of Ordnance Research.

†† Supported in part by the National Science Foundation, and in part by the U. S. Atomic Energy Commission.

* Work done while a visitor at the Lawrence Radiation Laboratory, University of California, Berkeley, California.

very small change in the ground-state energy of nuclear matter. It is the point of this note to show that this conclusion is in fact correct, the demonstration proceeding by use of the separation method[7] for evaluating the energy of nuclear matter.

Confining our attention to the interaction of particles with total-momentum zero, we see that hole-hole interactions may be included by replacing the Bethe-Goldstone equation[8] with[1,2]

$$(E - T_1 - T_2)\psi = (Q - P)\, V\, \psi \ , \tag{1}$$

where Q is an operator which projects both particles outside the Fermi sea, and P is an operator which projects both particles inside the sea.

$$\begin{aligned} \text{Thus } Q - P &= 1 && \text{if both particles are outside the Fermi sea,} \\ &= 0 && \text{if one particle is outside and one inside,} \\ &= -1 && \text{if both particles are inside the Fermi sea.} \end{aligned}$$

Let ϕ represent the wave function for a degenerate Fermi gas at a density appropriate to that of nuclear matter. Then the energy shift ΔE between the energy of the ideal gas and the interacting system is, in Brueckner theory[4],

$$\Delta E = \langle \phi \mid t \mid \phi \rangle \ , \tag{2}$$

where, using the separation method[7] and the modification of eq. (1) (Q replaced by $Q - P$), one obtains for t the series

$$t = (t_s + V_\ell) + (t_s + V_\ell)\, \frac{Q - P}{e}\, (t_s + V_\ell) - t_s\, \frac{1}{e_0}\, t_s \tag{3}$$

The notation is the same as reference 7, and the small contribution of higher-order terms has been discussed in this same reference. We may write eq. (3) as

$$t = t_0 + \Delta t \; , \tag{4}$$

where t_0 is the usual t matrix evaluated in the absence of hole-hole interactions (cf. Ref. 7), and Δt is the correction due to hole-hole interactions,

$$\Delta t = -(t_s + V_\ell) \frac{P}{e} (t_s + V_\ell) \; . \tag{5}$$

Thus the solutions to the modified Bethe-Goldstone equation are quite different from the solutions of the usual Bethe-Goldstone equation. This difference, which might confuse numerical evaluation of the energy, is, however, predominantly of such a character as to cancel when the total ground-state energy is evaluated.

Indeed the second-order perturbation term for a conventional interaction $\tilde{V}$ is

$$\tilde{V} \frac{P}{e} \tilde{V} \; , \tag{6}$$

which vanishes when summed over all states in the Fermi sea[2], as a consequence of the hermiticity of $\tilde{V}$. However, this cancellation does not occur in our case, where $\tilde{V}$ must be replaced by $t_s + V_\ell$. The long-range interaction V_ℓ is not quite hermitian, since the cutoff distance depends on momentum, while the short-range interaction t_s is, in fact, nearly antihermitian. The sum is also not hermitian, though more nearly so than V_ℓ alone. These results are illustrated in Table I.

TABLE I

Typical matrix elements of long- and short-range interactions (units Mev-Fermi^3) for standard potential of Reference 7.

	$k = 0.4\ f^{-1}$, $k' = 1.2\ f^{-1}$	$k = 1.2\ f^{-1}$, $k' = 0.4\ f^{-1}$
$(V_\ell)_{k'k}$	-357.7	-288.3
$(t_s)_{k'k}$	$+45.2$	-53.0
$(t_s + V_\ell)_{k'k}$	-312.5	-341.3

A crude calculation of the total contribution of the second-order hole-hole term (eq.(5)) to the ground-state energy yields, in view of the cancellation commented upon above, only about $+1/2$ Mev per particle.

This result is dependent upon the rapid convergence of the separation method, which is certainly sufficient for the purpose of this note. The general problem of convergence has been discussed in reference 7, whereas the convergence rate when hole-hole interactions are included is not materially altered. It should be observed that the smallness of the correction to the ground-state energy is intimately related to the smallness of the exclusion-principle contribution in the usual theory. In fact, these two terms are comparable in magnitude, and a quantitative study of the exclusion principle contribution in nuclear matter should properly proceed from the equation including hole-hole interactions. We conclude that even in the presence of singular potentials, hole-hole interactions do not significantly affect the ground-state energy of nuclear matter, and hence leave unaltered the quantitative results of Brueckner and Gammel.

FOOTNOTES

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